Sequential Screening with Effective Type-Enhancing Investment*

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Abstract

Due to the well-known efficiency–rent extraction trade-off, the principal-optimal mechanism in a pure screening environment (e.g., revenue maximization in auctions or cost minimization in procurements) typically entails distortion from the efficient allocation when agents possess private information at the time of contracting. In this paper, we introduce first-stage type-enhancing hidden investment with linear cost to a standard sequential screening model of procurement, and find that cost minimization must require social efficiency when the investment is sufficiently effective.

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1 Introduction

A central insight in the screening literature is that when agents possess private information at the time of contracting, the cost-minimizing allocation distorts from the efficient level (only no distortion at the "top"), due to the long-recognized efficiency–rent extraction trade-off. In a typical environment of sequential screening—e.g., the two-stage settings of Courty and Li [3] and Eső and Szentes [4]—agents are endowed at the time of contracting with a private type (the first-stage type), which determines the distribution of their second-stage type. As such, the central insight implies that at the optimum, the second-stage allocation is in general discriminatory over the first-stage type to optimally balance information elicitation and rent extraction in a dynamic setting.

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This paper shows that by introducing (natural) first-stage, type-enhancing hidden investment with linear cost to the standard sequential screening model, the cost-minimizing mechanism can, surprisingly, maximize social welfare. As a result, there is no allocative efficiency loss or distortion of investment (compared to first best). One practical motivation for such type-enhancing investment comes from the observation that situations are abundant in which agents can conduct hidden actions in the first stage to gain a better distribution of the second-stage type. For example, in a procurement setting, the supplier can conduct R&D to improve his chance of discovering a more cost-efficient way to deliver the product procured. The level of R&D investment of the agent (the supplier) is usually not observed by the principal (the procurer).

We demonstrate our result by considering a two-stage procurement model with a procurer (she, the principal) and a supplier (he, the agent). The procurer wishes to procure a product from the supplier, which she can acquire from an outside option at cost c_0. In the first stage, the agent does not know exactly his cost of producing the product. He is rather endowed with a capacity (i.e., his first-stage private type) that determines the distribution of his production cost in the second stage (i.e., his second-stage type). The agent's first-stage type is his private information, which can be determined by, for example, the agent's lab capacity, quality of facilities, or technology endowment. A higher type means a better distribution of production costs in the sense of first-order stochastic dominance. The agent can make an optimal level of unobservable investment to improve his first-stage type (i.e., increase his capacity) so that his second-stage production cost will be drawn from an improved ex post distribution. The contract is offered at the first stage, and the type-enhancing investment is conducted by the agent after the contract is accepted. At the second stage, the production cost is drawn from an ex post distribution determined by the improved type; again, the realized production cost is the agent's private information. The principal's objective is to design a contract to minimize her expected procurement cost.

We show that when the (constant) marginal cost of exerting effort[1] is relatively small, the cost-minimizing mechanism is exactly the efficient mechanism. As such, the optimal mechanism is a static one that only relies on the second-stage type—the benefit of sequential screening is completely absent, so the principal loses the power of screening at the first stage; more importantly, the optimal mechanism is ex post efficient. The main driving force for this surprising result is the *absence* of the usual efficiency–rent extraction trade-off in this environment. More precisely, when all first-stage types have incentive to exert effort, each first-stage type's information rent is *independent* of the

[1] We use "effort" and "investment" interchangeably in this paper.

allocation rule. Thus, moving the allocation rule toward the efficient level will not alter any first-stage type's information rent. Such independence results from the fact that the agent's information rent purely comes from the investment cost he saves when misreporting his first-stage type.

Our paper contributes to the literature on dynamic mechanism design, which originates in the seminal work of Baron and Besanko [1]. Courty and Li [3] and Eső and Szentes [4] demonstrate in different environments on sequential screening with pure adverse selection that at the optimum, the second-stage mechanism is discriminatory across first-stage types. Krähmer and Strausz [6] introduce endogenous information acquisition to the monopolistic price discrimination model of Courty and Li [3]. Pavan, Segal, and Toikka [12] provide a general treatment of optimal dynamic mechanism design with pure adverse selection. Li and Shi [8] and Guo, Li, and Shi [5] study discriminatory information disclosure in sequential screening with pure adverse selection. Our paper contributes to this strand of literature by studying a mixed adverse selection and moral hazard problem in a dynamic environment, in which the agent can make an investment in the first stage to change the distribution of his second-stage type.

Our paper essentially introduces moral hazard in the first stage to the seminal work of Courty and Li [3] on sequential screening. Courty and Li [3] study monopolistic pricing discrimination, in which consumers only know the distribution of their valuations at the time of contracting and then learn the actual valuations in the second stage. We adopt a parallel procurement setting assuming that at the time of contracting, the agent only knows the distribution of his production cost and subsequently discovers his actual cost. However, our setting departs from Courty and Li [3] by further taking into account the possibility that the agent can also make some unobservable investment to improve his first-stage type. One insight in Courty and Li [3] is that at the optimum, the second-stage mechanism discriminates over first-stage types. Such discrimination yields efficiency loss in the second stage. We show that the efficiency can be completely restored by introducing moral hazard. This result differs from the distortive allocation result in Liu and Lu [9], who also introduce moral hazard to a sequential screening model. The key difference is that in Liu and Lu [9] the first-stage type is the marginal cost of exerting effort, and the effort fully determines the distribution of the second-stage type. Therefore, the agent's information rent depends on the second-stage allocation rules (unless, trivially, all types are induced to exert zero effort), so the usual efficiency–rent extraction trade-off is still present there. However, here, the key driving force for our result is the absence of such a trade-off.

In classical sequential screening settings but with ex post participation constraints, Krähmer and Strausz [7] and Bergemann, Castro, and Weintraub [2] demonstrate that under some conditions, the optimal contract is a static one that only screens the agent's second-stage type. Our result suggests another possibility for such independence of the first-stage type at the optimum: When allowing the possibility of a type-enhancing hidden action, the optimal contract can also only respond to the agent's second-stage type. However, since in our paper the incentive to exert effort differs between first-stage types, the first-stage expected payoff also varies across first-stage types.

The rest of the paper is organized as follows. Section 2 sets up the model. We study the first-best benchmark in Section 3. Section 4 presents the main analysis, and Section 5 discusses the results and concludes. The appendix collects some technical proofs.

2 The Model

A risk-neutral buyer (the principal, she) wishes to procure a product from a risk-neutral supplier (the agent, he). At the time of contracting, the agent does not know his exact cost of supplying the product. However, the agent has some private signal, θ, which determines his future production efficiency. Specifically, the production cost c is randomly drawn from a cumulative distribution function $H(\cdot,\theta)$ (parameterized by θ) whose support is $[\underline{c},\overline{c}]$ with $0 \leq \underline{c} < \overline{c} \leq \infty$. Roughly speaking, higher θ means that the agent would have a higher expected efficiency in delivering the product. The exact meaning will become clear soon. From the principal's perspective, θ is randomly drawn from a CDF $G(\cdot)$ with density function $g(\cdot) > 0$ everywhere over the support $[\underline{\theta},\overline{\theta}]$, where $\underline{\theta} \geq 0$.

Departing from the classical sequential screening setting, here the agent can make unobservable investment $\alpha \geq 0$ at the cost of $\gamma_0\alpha$ to increase his type from θ to $\theta+\alpha$, where γ_0 is a positive constant. That is, with investment α, the production cost c will be drawn from distribution $H(\cdot,\theta+\alpha)$. The cost of producing the product, which is privately observed by the agent, is realized after the investment. The agent's delivery cost c is incurred only when the principal acquires the product from him. When the trade does not occur between the principal and the agent, the principal exercises her outside option for the product at cost c_0 with $c_0 \in (\underline{c},\overline{c})$. γ_0, $G(\cdot)$, $H(\cdot,\cdot)$, and c_0 are public information. The principal's objective is to minimize her expected procurement cost.

The timing of the game is as follows.

Time 0: The agent is privately informed about his type θ.

Time 1: The principal offers a contract and she commits to it. If the agent rejects, then the game ends and he obtains his reservation utility, which is normalized as zero. If the agent accepts, he decides how much investment α to make to improve his first-stage type.

Time 2: The agent's private delivery cost is drawn from $H(\cdot, \theta + \alpha)$. The contract is then executed.

Following the revelation principle (Myerson [11]), we restrict to direct mechanisms $\{p(\theta, c), y(\theta, c), \alpha(\theta)\}_\theta$, in which θ and c are the agent's first-stage and second-stage reports, respectively; here $p(\theta, c)$ is the acquisition probability, $y(\theta, c)$ is the payment to the agent, and $\alpha(\theta)$ is the principal's recommendation of effort to the agent. More details can be found in Section 4.

We assume that $H(c, z)$ is twice continuously differentiable in $(c, z) \in [\underline{c}, \overline{c}] \times [\underline{\theta}, +\infty)$ and that the density function $h(c, z)$ (i.e., $H_1(c, z)$) is strictly positive everywhere over the support.[2] In addition,

$$H_2(c, z) > 0, H_{22}(c, z) < 0, \forall c \in (\underline{c}, \overline{c}).$$

Positive $H_2(c, z)$ means that higher z leads to a better cost distribution in the sense of first-order stochastic dominance; this is exactly what we mean by saying "a higher θ means a better expected production efficiency" at the beginning of this section. Negative $H_{22}(c, z)$ means that the marginal effect of z decreases.[3] Our formulation of $H(c, z)$ covers the following widely adopted CDF as a special case: $H(c, z) = 1 - (1 - F(c))^{z+\beta_0}$, where $F(c)$ is a CDF with strictly positive density function everywhere over the support $[\underline{c}, \overline{c}]$, and $\beta_0 > 0$ is a constant.

We make the following regularity assumption, as in the dynamic mechanism design literature.

Assumption 1 (Regularity)**.** *The virtual cost function* $J(c, \theta) = c + \frac{1-G(\theta)}{g(\theta)} \frac{H_2(c,\theta)}{h(c,\theta)}$ *is strictly increasing in* c *when* $c \in [\underline{c}, c_0]$ *and strictly decreasing in* θ*.*

Assumption 1 is satisfied when, for example, 1) the hazard rate $\frac{g(\theta)}{1-G(\theta)}$ is increasing in θ; 2) $H_2(c, \theta)/h(c, \theta)$ is increasing in c; and 3) $H_2(c, \theta)/h(c, \theta)$ is decreasing in θ. The monotone hazard rate assumption is standard in the mechanism design literature, and the latter two assumptions

[2] For a two-variable function, we use subscripts 1 and 2 to represent the partial derivative with respect to the first and second argument, respectively.

[3] This assumption validates the "first-order approach," which replaces the agent's moral hazard incentive compatibility with the first-order condition.

parallel Assumption 1 and Assumption 2, respectively, in Eső and Szentes [4].

Note that when no investment is allowed—i.e., the agent's first-stage type is θ and his second-stage type c is a random draw from $H(\cdot, \theta)$—our setting reduces to a standard sequential screening setting, as in Courty and Li [3] and Eső and Szentes [4]. Given Assumption 1, standard arguments, as in these two papers,[4] imply that the cost-minimizing mechanism is deterministic and has the feature that for each θ, the trade happens if and only if the virtual cost $J(c, \theta) \le c_0$. In other words, the trade happens if and only if the realized second-stage type $c \le c_B(\theta)$, where $c_B(\theta) \in (\underline{c}, \overline{c})$ is uniquely characterized by

$$J(c_B(\theta), \theta) - c_0 = 0. \tag{1}$$

It is easy to see that $c_B(\theta)$ is strictly increasing in θ and that $c_B(\theta) \le c_0$ with equality only when $\theta = \overline{\theta}$. Therefore, when no investment is allowed, the optimal allocation distorts from the efficient level c_0, leading to efficiency loss.

We further make the following assumption that the marginal cost of investment is relatively small.

Assumption 2 (Effective Investment Technology)**.**

$$\gamma_0 < \min_{\theta \in [\underline{\theta}, \overline{\theta}]} \int_{\underline{c}}^{c_B(\theta)} H_2(c, \theta) dc.$$

Note that $\min_{\theta \in [\underline{\theta}, \overline{\theta}]} \int_{\underline{c}}^{c_B(\theta)} H_2(c, \theta) dc$ is some fixed, strictly positive number, so Assumption 2 is satisfied when γ_0 is small enough. Assumption 2 guarantees that all θ types have strong incentive to exert effort, which, as will become clear later, implies the dominance of efficiency effect over rent extraction effect. Our main result is the following.

Theorem. *Under Assumptions 1 and 2, the cost-minimizing mechanism is ex post efficient and therefore coincides with the efficient mechanism.*

3 The First-best Benchmark

We first study the first-best benchmark in which the agent's types and his effort level are public information. Suppose that for the agent with type θ and realized cost c, the contract specifies the

[4] The proof is available from the authors upon request.

payment to the agent $y^{FB}(\theta, c)$ and the acquisition probability $p^{FB}(\theta, c)$. In the first stage, the contract prescribes the agent's investment level $\alpha^{FB}(\theta)$.

The social cost for type θ is

$$\gamma_0 \alpha^{FB}(\theta) + \int_{\underline{c}}^{\overline{c}} [p^{FB}(\theta, c)c + (1 - p^{FB}(\theta, c))c_0] h(c, \theta + \alpha^{FB}(\theta)) dc. \tag{2}$$

It is obvious that the first-best allocation is ex post efficient:

$$p^{FB}(\theta, c) = \begin{cases} 1, \text{ when } c \leq c_0 \\ 0, \text{ when } c > c_0 \end{cases}.$$

Thus, the total cost further boils down to

$$\gamma_0 \alpha^{FB}(\theta) - \int_{\underline{c}}^{c_0} H(c, \theta + \alpha^{FB}(\theta)) dc + c_0. \tag{3}$$

Under Assumption 2, $\alpha^{FB}(\theta) > 0$ and satisfies

$$\int_{\underline{c}}^{c_0} H_2(c, \theta + \alpha^{FB}(\theta)) dc = \gamma_0. \tag{4}$$

Let $\theta^* > \overline{\theta}$ be the unique solution to $\int_{\underline{c}}^{c_0} H_2(c, \theta^*) dc = \gamma_0$. Thus,

$$\alpha^{FB}(\theta) = \theta^* - \theta.$$

This is intuitive. In fact, (4) says that the first-best mechanism equates the marginal benefit of investment, which only depends on the "effective" type $\theta + \alpha^{FB}(\theta)$, to the marginal cost of investment, which is always a constant γ_0. Therefore, the effective type $\theta + \alpha^{FB}(\theta)$ must be a constant. The following proposition summarizes the main properties of the first-best mechanism.

Proposition 1. *The first-best allocation is ex post efficient. The first-best investment of type θ, $\alpha^{FB}(\theta) = \theta^* - \theta$, is strictly positive and strictly decreasing in θ. However, the effective first-stage type $\theta + \alpha^{FB}(\theta)$ is a constant θ^*.*

4 Analysis of the Cost-minimizing Mechanism

Now we turn to the original problem. There is no loss of generality to focus on truthful direct mechanisms, according to Myerson [11]. In the first stage, when the agent reports $\hat{\theta}$, he receives an investment recommendation $\alpha(\hat{\theta}) \geq 0$. The agent decides on investment level α after reporting $\hat{\theta}$. In the second stage, his delivery cost c is realized according to $H(\cdot, \theta+\alpha)$, where θ is his true first-stage type; he further reports his cost realization. The report is denoted by $\hat{c}$. Then the payment rule $y(\hat{\theta}, \hat{c})$ and the acquisition probability $p(\hat{\theta}, \hat{c})$ are executed.

4.1 Stage Two

Assuming truthfully reported θ in stage one, suppose that the agent's true provision cost is c, but he reports $\hat{c}$. Let $\tilde{\pi}(\theta, \hat{c}, c)$ be his expected payoff in stage two. Then

$$\tilde{\pi}(\theta, \hat{c}, c) = y(\theta, \hat{c}) - p(\theta, \hat{c})c. \tag{5}$$

Envelope Theorem yields

$$\frac{d\tilde{\pi}(\theta, c, c)}{dc} = \frac{\partial \tilde{\pi}(\theta, \hat{c}, c)}{\partial c}|_{\hat{c}=c} = -p(\theta, c),$$

which leads to

$$\tilde{\pi}(\theta, c, c) = \tilde{\pi}(\theta, \overline{c}, \overline{c}) + \int_c^{\overline{c}} p(\theta, s)ds, \forall \theta. \tag{6}$$

It is clear that the second-stage incentive compatibility (IC) is equivalent to that (6) holds and that $p(\theta, c)$ is decreasing in c for any fixed θ. Note that if the agent misreported his type in stage one as $\hat{\theta}$, he will still truthfully report c in stage two. This is because the agent's first-stage type does not directly enter his second-stage payoff.

4.2 Stage One

The first-stage IC requires that the agent report his type truthfully and follow the principal's recommendation on investment level. Note that when the agent reports his type and then receives the recommendation (which depends on the report), he always chooses a unique optimal effort level regardless of the recommendation he receives. This is because the agent's belief is not affected by the recommendation and, as we will show in the derivation of the moral hazard constraint (9) (in the Appendix), the agent's payoff is (strictly) concave in effort, so that he will not randomize his effort level. Such an effort level only depends on his true type and the type he reported to the principal.

If the agent with type θ reports $\hat{\theta}$ and invests α, his expected payoff is

$$\hat{\pi}(\alpha, \hat{\theta}, \theta) = -\gamma_0 \alpha + \int_{\underline{c}}^{\overline{c}} \tilde{\pi}(\hat{\theta}, c, c) h(c, \theta + \alpha) dc.$$

In the derivation of the moral hazard constraint (9) (in the Appendix), we will show that $\hat{\pi}(\alpha, \hat{\theta}, \theta)$ is (strictly) concave in α. Let $\alpha(\hat{\theta}, \theta) = \arg\max_{\alpha \geq 0} \hat{\pi}(\alpha, \hat{\theta}, \theta)$,[5] and $\pi(\hat{\theta}, \theta) = \max_{\alpha \geq 0} \hat{\pi}(\alpha, \hat{\theta}, \theta) = \hat{\pi}(\alpha(\hat{\theta}, \theta), \hat{\theta}, \theta)$, which is the agent's expected utility when his true type is θ but he reports $\hat{\theta}$, given that he will respond optimally by taking $\alpha(\hat{\theta}, \theta)$ when receiving the recommendation $\alpha(\hat{\theta})$. The first-stage IC then requires that

$$\pi(\theta, \theta) \geq \pi(\hat{\theta}, \theta), \forall \theta, \hat{\theta}, \tag{7}$$

which we call the IC_1 constraint. Note that $\alpha(\hat{\theta}, \theta)$ is determined as follows:[6]

$$-\gamma_0 + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta}, c) H_2(c, \theta + \alpha(\hat{\theta}, \theta)) dc \leq 0, \text{ with equality if } \alpha(\hat{\theta}, \theta) > 0, \forall \theta, \hat{\theta}. \tag{8}$$

Note that from the above equation, one has $\alpha(\hat{\theta}, \theta) = \max\{\delta(\hat{\theta}) - \theta, 0\}$, where $\delta(\hat{\theta}) \geq \underline{\theta}$, corresponding to $p(\hat{\theta}, c)$, is the unique value such that $\int_{\underline{c}}^{\overline{c}} p(\hat{\theta}, c) H_2(c, \delta(\hat{\theta})) dc = \gamma_0$.[7] This is because the marginal cost of investment is always a constant γ_0, whereas the benefit is (strictly) concave in investment. However, notice that $\delta(\hat{\theta})$ is endogenous, which depends on $\hat{\theta}$ and the allocation rule $p(\hat{\theta}, c)$.

When the type θ agent reports truthfully, the principal's recommendation $\alpha(\theta)$ must coincide with the agent's optimal effort choice (obedience), i.e., $\alpha(\theta) = \alpha(\theta, \theta)$. This is characterized by the following first-order condition:

$$-\gamma_0 + \int_{\underline{c}}^{\overline{c}} p(\theta, c) H_2(c, \theta + \alpha(\theta)) dc \leq 0, \text{ with equality if } \alpha(\theta) > 0, \forall \theta, \tag{9}$$

which we call the moral hazard constraint MHC. IC_1 and MHC constitute the first-stage IC

[5] $\alpha(\hat{\theta}, \theta)$ exists because $\lim_{\alpha \to +\infty} \hat{\pi}(\alpha, \hat{\theta}, \theta) = -\infty$.

[6] Details can be found in (16).

[7] When $\int_{\underline{c}}^{\overline{c}} p(\hat{\theta}, c) H_2(c, \underline{\theta}) dc < \gamma_0$, $\delta(\hat{\theta})$ does not exist. In this case, without loss of generality, define $\delta(\hat{\theta}) = 0$.

constraints. The proof of MHC (9) is in the Appendix.

Given (8), we obtain the following necessary conditions for the first-stage IC.

Lemma 1. *The first-stage IC constraint implies*

i) Envelope condition:

$$\pi(\theta,\theta)=\pi(\underline{\theta},\underline{\theta})+\int_{\underline{\theta}}^{\theta}\int_{\underline{c}}^{\overline{c}}p(s,c)H_2(c,s+\alpha(s))dcds,\ \forall\theta.$$

ii) The recommendation $\alpha(\theta)$ satisfies the moral hazard constraint MHC (9).

4.3 The Principal's Problem

By Lemma 1, the expected cost of procurement, which is the sum of the expected social cost and the agent's expected utility, can be written as

$$TC=\int_{\underline{\theta}}^{\overline{\theta}}\left\{\gamma_0\alpha(\theta)+\int_{\underline{c}}^{\overline{c}}p(\theta,c)\left[c+\frac{1-G(\theta)}{g(\theta)}\frac{H_2(c,\theta+\alpha(\theta))}{h(c,\theta+\alpha(\theta))}-c_0\right]h(c,\theta+\alpha(\theta))dc\right\}g(\theta)d\theta+\pi(\underline{\theta},\underline{\theta})+c_0.$$

The term in the square bracket,

$$c+\frac{1-G(\theta)}{g(\theta)}\frac{H_2(c,\theta+\alpha(\theta))}{h(c,\theta+\alpha(\theta))},$$

is the adjusted virtual cost, denoted as $\tilde{J}(c,\theta,\theta+\alpha(\theta))$, with $\theta+\alpha(\theta)$ as the "effective first-stage type," which resembles the standard virtual cost in the classical sequential screening model.

The principal's problem can be expressed as

$$\min_{\{\alpha(\theta)\geq 0,\ p(\theta,c)\in[0,1]\}} TC$$

subject to

$$\text{constraints } IC_1 \text{ (7) and } MHC \text{ (9);} \tag{10}$$

$$\pi(\theta,\theta)\geq 0,\ \forall\theta; \tag{11}$$

$$\tilde{\pi}(\theta,c,c)=\tilde{\pi}(\theta,\overline{c},\overline{c})+\int_{c}^{\overline{c}}p(\theta,s)ds, \forall c, \forall\theta; \tag{12}$$

$$p(\theta,c) \text{ is decreasing in } c \text{ for any } \theta. \tag{13}$$

(10) is the first-stage IC constraint; (11) is the first-stage IR constraint, and (12) and (13) are the equivalent conditions for the second-stage IC constraint. We call this Problem (O).

Proposition 2. *The optimal solution to Problem (O) is*

$$p^*(\theta, c) = \begin{cases} 1, \text{ if } c \leq c_0 \\ 0, \text{ if } c > c_0 \end{cases}, \quad a^*(\theta) = a^{FB}(\theta) = \theta^* - \theta, \ \forall \theta.$$

Thus, at the optimum, the second stage is efficient and the investment level of each type is the same as the first-best one. Since the optimal allocation rule is ex post efficient, the agent's incentive to exert effort coincides with the efficiency-maximizing one. This immediately implies the following result.

Corollary 1. *Under Assumptions 1 and 2, the cost-minimizing mechanism is both ex ante and ex post efficient.*

5 Discussion

5.1 Why Does It Maximize Social Welfare?

A typical result from the sequential screening literature is that the second-stage allocation is discriminatory over first-stage types. However, here, by introducing hidden actions to sequential screening settings, the principal completely loses the power of dynamic screening, as the optimal contract responds only to the agent's second-stage type. In classical sequential screening settings, but with ex post participation constraints, Krähmer and Strausz [7] and Bergemann, Castro, and Weintraub [2] demonstrate that under some conditions, the optimal contract is a static one, which only screens the second-stage type. Our result suggests another possibility: When allowing the possibility of a first-stage, type-enhancing hidden action, the optimal two-stage contract can also only screen the second-stage type, though the contract still needs to be implemented in two stages. Nevertheless, since equilibrium investment differs across first-stage types, their interim payoffs also vary.[8]

More importantly, the optimal contract is both ex ante and ex post efficient, so there is no efficiency loss. This is striking, as classical insights tell us that there should be efficiency loss

[8] In fact, the first-stage expected payoff of type θ is $\gamma_0(\theta - \underline{\theta})$.

(distortion of allocative efficiency or effort) to overcome information asymmetry.

These results deserve careful investigation. The main driving forces are illustrated below in two points: the absence of the typical efficiency–rent extraction trade-off with positive investment and the dominance of efficient investment over zero investment.

1. *Absence of the typical efficiency–rent extraction trade-off.* For any incentive-compatible mechanism inducing positive investment for all θ, the usual trade-off between efficiency and rent extraction does not arise. In fact, in this case, the MHC (9) is binding for all θ, so that $\pi(\theta,\theta) = \pi(\underline{\theta},\underline{\theta}) + \gamma_0(\theta - \underline{\theta})$ from Lemma 1. The crucial observation is that the agent's information rent does not depend on the mechanism. This differs dramatically from the usual adverse-selection models, in which the agent's information rent typically relies on the allocation rule, so there is a trade-off between efficiency and rent extraction. Thus, focusing on mechanisms that induce positive effort for all types, the rent extraction effect is completely absent. Under Assumption 2, the efficient allocation $p^*(\theta, c)$ belongs to such a class of mechanisms. It follows directly that the only effect left—the efficiency effect—is just the social cost, whose optimum must then be exactly the same as the efficient mechanism.[9]

This result can also be understood from another angle. The principal's payoff is the sum of the social cost and the agent's information rent. Obviously, the efficient mechanism maximizes the social welfare (i.e., minimizes the social cost). Thus, if the efficient mechanism also minimizes the agent's information rent, it must be optimal. The above paragraph explains that this is exactly the case within the class of mechanisms that induce positive effort for all types—in fact, all mechanisms in this class lead to the same information rents: $\pi(\theta,\theta) = \pi(\underline{\theta},\underline{\theta}) + \gamma_0(\theta - \underline{\theta})$ for type θ.

It is helpful to further explain why the information rent does not depend on the mechanism in this case.[10] Consider type θ's incentive to deviate to type $\hat{\theta} < \theta$. Here $\hat{\theta}$ is very close to θ such that after deviation, type θ still invests. By the discussion after (8), $\alpha(\hat{\theta},\theta) = \delta(\hat{\theta}) - \theta$, which is implied by the fact that the marginal cost of exerting effort is constant and the benefit of exerting

[9] Mathematically, the total cost can be decomposed into

$$TC = \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \underbrace{\gamma_0 \alpha(\theta) + \int_{\underline{c}}^{\overline{c}} p(\theta, c)(c - c_0) h(c, \theta + \alpha(\theta)) dc + c_0}_{\text{social cost}} + \underbrace{\pi(\underline{\theta},\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \int_{\underline{c}}^{\overline{c}} p(s,c) H_2(c, s + \alpha(s)) dc ds}_{\text{rent}=\pi(\underline{\theta},\underline{\theta})+\gamma_0(\theta-\underline{\theta})} \right\} g(\theta) d\theta,$$

so that the only effect left is the social cost, which is exactly the same as (2).

[10] Strictly speaking, it is the difference between any two θ types' information rents that is independent of the mechanism. The information rent of type $\underline{\theta}$ still depends on the mechanism. However, as usual, it must be 0 at the optimum.

effort (i.e., the expected payoff from the second stage) only relies on the effective first-stage type $\theta + \alpha(\hat{\theta}, \theta)$. The agent's payoff consists of the cost of investment and the benefit of investment. The difference between the deviation payoff for type θ, $\pi(\hat{\theta}, \theta)$, and the equilibrium payoff of type $\hat{\theta}$, $\pi(\hat{\theta}, \hat{\theta})$, is only the cost type θ saves from exerting effort, which is $\gamma_0(\theta - \hat{\theta})$. To see this, notice that the benefit of investment solely depends on $\delta(\hat{\theta})$; thus these two types' benefits must be the same. Therefore, the advantage of type θ over type $\hat{\theta}$ is the cost saved when both try to achieve the effective first-stage type $\delta(\hat{\theta})$ by making investment, which is $\gamma_0(\alpha(\hat{\theta}, \hat{\theta}) - \alpha(\hat{\theta}, \theta)) = \gamma_0(\theta - \hat{\theta})$. Therefore, IC implies that $\pi(\theta, \theta) \geq \pi(\hat{\theta}, \theta) = \pi(\hat{\theta}, \hat{\theta}) + \gamma_0(\theta - \hat{\theta})$. Reversing the roles of θ and $\hat{\theta}$, we also have $\pi(\hat{\theta}, \hat{\theta}) \geq \pi(\theta, \theta) + \gamma_0(\hat{\theta} - \theta)$. Thus, it must be the case that $\pi(\theta, \theta) - \pi(\hat{\theta}, \hat{\theta}) = \gamma_0(\theta - \hat{\theta})$, which is independent of the mechanism.

The absence of the typical efficiency–rent extraction trade-off when the induced investment is positive immediately implies that the efficient mechanism that maximizes social welfare must dominate any other incentive-compatible mechanism that induces positive investment for every type. It remains to explain why zero investment cannot be optimal, which is covered in the next point. Note that although the efficient mechanism minimizes the social cost within all feasible mechanisms, it does not necessarily minimize the agent's information rent;[11] thus, it is unclear whether the efficient mechanism is optimal. In fact, when the MHC (9) is slack for some positive measure of types, from Lemma 1, $\pi(\theta, \theta) < \pi(\underline{\theta}, \underline{\theta}) + \gamma_0(\theta - \underline{\theta})$ when θ is within some neighborhood of $\bar{\theta}$, leading to a lower information rent than the efficient mechanism.

2. *Dominance of the efficient mechanism over zero-investment mechanisms.* Assumption 2 is crucial for such dominance. Intuitively, Assumption 2 guarantees that the marginal cost of exerting effort is small enough so that zero investment is never desired. To see why Assumption 2 takes that form, we need to link the current mechanism to classical sequential screening settings without actions (i.e., investment must be zero). We have seen that focusing on mechanisms that induce positive investment for all types, the cost-minimizing mechanism is exactly the efficient mechanism. What about mechanisms that induce zero investment for some type: Can they be better than the efficient mechanism in terms of total cost? For convenience, call such a situation a *zero-investment situation.*

In the zero-investment situation, the usual efficiency–rent extraction trade-off arises. We claim that in this case, Assumption 2 implies that the efficiency effect dominates the rent extraction

[11] It minimizes the information rent when restricting to the class of mechanisms that induce positive effort for all types.

effect, so the principal should try to make the mechanism as efficient as possible. Perhaps the dominance can be better explained by focusing on deterministic mechanisms and using the adjusted virtual cost $\tilde{J}(c, \theta, \theta + \alpha(\theta))$. For deterministic mechanisms, each first-stage type θ is assigned a second-stage cutoff $c(\theta)$ such that when the reported $c \leq c(\theta)$, trade happens with probability one; otherwise, trade happens with probability zero. Suppose that type $\tilde{\theta}$ makes no investment in the zero-investment situation. In fact, for type $\tilde{\theta}$, its adjusted virtual cost coincides with the virtual cost $J(c, \tilde{\theta})$ in the sequential screening model without actions. We have seen that the cutoff $c_B(\tilde{\theta})$, identified in (1), is the best way to balance efficiency and rent extraction, as it minimizes the expected virtual cost $\mathbf{E}[J(c, \tilde{\theta}) - c_0]$. However, this is not feasible because under Assumption 2, type $\tilde{\theta}$ actually exerts effort. Now, under Assumption 2, the threshold cutoff $\tilde{c}(\tilde{\theta})$ such that type $\tilde{\theta}$ exerts zero effort—i.e., $\int_{\underline{c}}^{\tilde{c}(\tilde{\theta})} H_2(c, \tilde{\theta}) dc = \gamma_0$—is strictly less than $c_B(\tilde{\theta})$. Thus, when the mechanism induces zero investment for $\tilde{\theta}$, one must have $c(\tilde{\theta}) \leq \tilde{c}(\tilde{\theta}) \leq c_B(\tilde{\theta})$. Since the expected virtual cost $\mathbf{E}[J(c, \tilde{\theta}) - c_0]$ becomes smaller when the second-stage allocation cutoff is closer to $c_B(\tilde{\theta})$, when $c(\tilde{\theta}) < \tilde{c}(\tilde{\theta})$, increasing the cutoff for type $\tilde{\theta}$ from $c(\tilde{\theta})$ to $\tilde{c}(\tilde{\theta})$ always induces zero effort and lowers the expected virtual cost. Thus, ignoring IC, the principal should set $c(\theta) = \tilde{c}(\theta)$ for all θ types exerting zero effort. However, when the cutoff is $\tilde{c}(\theta)$ for all θ types exerting zero effort, it goes back to the previous point at which the MHC (9) is binding, which is dominated by the efficient mechanism.

5.2 Nonlinear Cost Functions

As mentioned before, a typical result from the sequential screening literature is that the second-stage allocation is discriminatory over first-stage types: There is a second-stage allocative distortion for each first-stage type (except the "top" type). Allowing a first-stage type-enhancing investment, our analysis demonstrates that with linear investment cost, the second-stage allocative distortion can disappear completely for all first-stage types. In fact, if the investment cost is convex (i.e., the cost of investing α is a function $C(\alpha)$, with $C(0) = 0$, $C' > 0$, and $C'' \geq 0$), the second-stage allocative distortion will be mitigated for all first-stage types—in other words, the second-stage allocative cutoff $c(\theta)$ will be (weakly) higher than the cutoff $c_B(\theta)$ without investment. This mitigation of distortion mainly results from the fact that with investment, the efficiency effect dominates the rent extraction effect in the efficiency–rent extraction tradeoff: Though increasing the cutoff leaves more information rents to the agent, it also induces more investment which moves towards the socially efficient level; moreover, the latter effect dominates the former one. Thus, the mitigation of distortion is a general phenomenon. Our main analysis sharply illustrates this by examining a linear-cost form that the distortion can disappear entirely.

5.3 Concluding Remarks

This paper introduces first-stage type-enhancing hidden investment with linear cost to the standard sequential screening model. In the standard sequential screening model, the second-stage mechanism often involves discrimination against less efficient first-stage types. However, we find that when the (constant) marginal cost of investment is small, the optimal mechanism maximizes social welfare, and the optimal mechanism only screens based on second-stage types. This result complements the conventional insight in the screening literature with pure adverse selection that efficiency must be sacrificed to reduce information rent at the optimum.

6 Appendix

Appendix A proves the results in the main text, and Appendix B provides proofs for the lemmas used in Appendix A.

6.1 Appendix A

Derivation of the moral hazard constraint (9): Since (6) holds for all reported type $\hat{\theta}$, we have

$$\begin{aligned}\hat{\pi}(\alpha,\hat{\theta},\theta) &= -\gamma_0\alpha + \tilde{\pi}(\hat{\theta},\bar{c},\bar{c}) + \int_{\underline{c}}^{\bar{c}}\left(\int_{c}^{\bar{c}} p(\hat{\theta},s)ds\right)h(c,\theta+\alpha)dc \\ &= -\gamma_0\alpha + \tilde{\pi}(\hat{\theta},\bar{c},\bar{c}) + \int_{\underline{c}}^{\bar{c}} p(\hat{\theta},c)H(c,\theta+\alpha)dc. \end{aligned} \tag{14}$$

Taking derivative with respect to α yields[12]

$$\frac{\partial\hat{\pi}(\alpha,\hat{\theta},\theta)}{\partial\alpha} = -\gamma_0 + \int_{\underline{c}}^{\bar{c}} p(\hat{\theta},c)H_2(c,\theta+\alpha)dc. \tag{15}$$

[12] The differentiablity is implied by the Lebesgue's dominated convergence theorem.

Second-order derivative

$$\frac{\partial^2 \hat{\pi}(\alpha,\hat{\theta},\theta)}{\partial \alpha^2} = \int_{\underline{c}}^{\overline{c}} p(\hat{\theta},c) H_{22}(c,\theta+\alpha) dc < 0,$$

when $p(\hat{\theta},c) > 0$ on a positive measure subset of $[\underline{c},\overline{c}]$. Since the agent's expected payoff $\hat{\pi}(\alpha,\hat{\theta},\theta)$ is (strictly) concave in α, the optimal α—denoted as $\alpha(\hat{\theta},\theta)$—is unique.[13] Thus, the agent with type θ who reports $\hat{\theta}$ will choose optimal effort level $\alpha(\hat{\theta},\theta)$, characterized by

$$-\gamma_0 + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta},c) H_2(c,\theta+\alpha(\hat{\theta},\theta)) dc \leq 0, \text{ with equality if } \alpha(\hat{\theta},\theta) > 0. \tag{16}$$

Now $\pi(\hat{\theta},\theta)$—the type θ agent's highest expected payoff when he reports $\hat{\theta}$—can be expressed as

$$\pi(\hat{\theta},\theta) = \hat{\pi}(\alpha(\hat{\theta},\theta),\hat{\theta},\theta) = -\gamma_0 \alpha(\hat{\theta},\theta) + \tilde{\pi}(\hat{\theta},\overline{c},\overline{c}) + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta},c) H(c,\theta+\alpha(\hat{\theta},\theta)) dc. \tag{17}$$

When the type θ agent reports truthfully, the principal's recommendation $\alpha(\theta)$ must coincide with the agent's optimal effort choice (obedience), so that $\alpha(\theta) = \alpha(\theta,\theta)$, leading to the *MHC* constraint (9). Since the agent's utility function $\hat{\pi}(\alpha,\hat{\theta},\theta)$ is strictly concave in α (> 0), the "first-order approach" is valid; we can replace the original incentive compatibility constraint for moral hazard with the above first-order condition. □

Proof of Lemma 1: The moral hazard constraint is obvious. Note that $\alpha(\hat{\theta},\theta)$ must be bounded, as $\lim_{\alpha\to+\infty} \hat{\pi}(\alpha,\hat{\theta},\theta) = -\infty$ from (14). The following lemma establishes useful properties of $\alpha(\hat{\theta},\theta)$ and $\pi(\hat{\theta},\theta)$.

Lemma A1. *For any $\hat{\theta}$, $\alpha(\hat{\theta},\cdot)$ is continuous. Moreover, it is differentiable everywhere except possibly at one point.*

Lemma A2. *For any $\hat{\theta}$, $\pi(\hat{\theta},\cdot)$ is continuously differentiable; moreover, $\frac{\partial \pi(\hat{\theta},\theta)}{\partial \theta} = \int_{\underline{c}}^{\overline{c}} p(\hat{\theta},c) H_2(c,\theta+\alpha(\hat{\theta},\theta)) dc$.*

The proofs of Lemmas A1 and A2 are relegated to Appendix B. Since $\pi(\hat{\theta},\theta)$ is continuously differentiable in θ over $[\underline{\theta},\overline{\theta}]$ by Lemma A2, it is Lipschitz continuous and thus absolutely continuous. Also note that the derivative $\frac{\partial \pi(\hat{\theta},\theta)}{\partial \theta}$ is bounded from Lemma A2. By envelope theorem (cf. Milgrom

[13] $\frac{\partial^2 \hat{\pi}(\alpha,\hat{\theta},\theta)}{\partial \alpha^2} = 0$ only when $p(\hat{\theta},c) = 0$ almost everywhere on $[\underline{c},\overline{c}]$. However, in this case, from the first-order condition we know that the agent will optimally choose $\alpha = 0$. Therefore, in all cases the optimal α is unique. Note that $\alpha(\hat{\theta},\theta)$ exists, as $\lim_{\alpha\to+\infty} \hat{\pi}(\alpha,\hat{\theta},\theta) = -\infty$ from (14).

and Segal [10]), we have

$$\frac{d\pi(\theta,\theta)}{d\theta} = \frac{\partial \pi(\hat{\theta},\theta)}{\partial \theta}|_{\hat{\theta}=\theta} = \int_{\underline{c}}^{\overline{c}} p(\theta,c)H_2(c,\theta+\alpha(\theta))dc,$$

so that

$$\pi(\theta,\theta) = \pi(\underline{\theta},\underline{\theta}) + \int_{\underline{\theta}}^{\theta}\int_{\underline{c}}^{\overline{c}} p(s,c)H_2(c,s+\alpha(s))dcds. \tag{18}$$

□

Proof of Proposition 2: From Lemma 1, the IR constraint (11) is equivalent to $\pi(\underline{\theta},\underline{\theta}) \geq 0$, which must be binding at the optimum. Note that we have incorporated the envelope condition in Lemma 1 and the second-stage envelope condition (12) to simplify the expression of TC. Thus, further dropping IC_1 (7), Problem (O) can be relaxed to the following Problem (O-R).

$$\min_{\{\alpha(\theta)\geq 0, p(\theta,c)\in[0,1]\}} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \gamma_0\alpha(\theta) + \int_{\underline{c}}^{\overline{c}} p(\theta,c)\left[\tilde{J}(c,\theta,\theta+\alpha(\theta)) - c_0\right] h(c,\theta+\alpha(\theta))dc \right\} g(\theta)d\theta + c_0 \tag{19}$$

subject to

$$\text{constraint (9);} \tag{20}$$

$$p(\theta,c) \text{ is decreasing in } c \text{ for any } \theta. \tag{21}$$

The solution to Problem (O) cannot be better than that to Problem (O-R). Thus, if the solution to Problem (O-R) is also feasible in Problem (O), such solution solves Problem (O). The following result characterizes the optimal solution to Problem (O-R), the proof of which is in Appendix B.

Lemma A3. *The optimal solution to Problem (O-R) is*

$$p^*(\theta,c) = \begin{cases} 1, \ if\ c \leq c_0 \\ 0, \ if\ c > c_0 \end{cases}, \ a^*(\theta) = a^{FB}(\theta), \ \forall\theta.$$

We only need to show that $\{p^*(\theta,c),\alpha^*(\theta)\}_\theta$ satisfies IC_1 (7) in Problem (O); all other constraints in Problem (O) are trivially satisfied. By (17), when type θ agent reports $\hat{\theta}$, his expected payoff is

$$\pi(\hat{\theta},\theta) = -\gamma_0\alpha(\hat{\theta},\theta) + \tilde{\pi}(\hat{\theta},\overline{c},\overline{c}) + \int_{\underline{c}}^{c_0} H(c,\theta+\alpha(\hat{\theta},\theta))dc.$$

From (16), $\alpha(\hat{\theta},\theta) = \theta^* - \theta = \alpha^{FB}(\theta)$, regardless of $\hat{\theta}$. Then, under $\{p^*(\theta,c), \alpha^*(\theta)\}_\theta$,

$$\pi(\hat{\theta},\theta) = -\gamma_0\alpha^{FB}(\theta) + \tilde{\pi}(\hat{\theta},\overline{c},\overline{c}) + \int_{\underline{c}}^{c_0} H(c,\theta+\alpha^{FB}(\theta))dc. \tag{22}$$

On the other hand, by Lemma 1,

$$\pi(\theta,\theta) = \int_{\underline{\theta}}^{\theta}\int_{\underline{c}}^{c_0} H_2(c, s+\alpha^{FB}(s))dcds = \gamma_0(\theta-\underline{\theta}).$$

So

$$-\gamma_0\alpha^{FB}(\theta) + \tilde{\pi}(\theta,\overline{c},\overline{c}) + \int_{\underline{c}}^{c_0} H(c,\theta+\alpha^{FB}(\theta))dc = \pi(\theta,\theta) = \gamma_0(\theta-\underline{\theta}),$$

which further implies that

$$\begin{aligned}\tilde{\pi}(\theta,\overline{c},\overline{c}) &= \gamma_0(\theta-\underline{\theta}) + \gamma_0\alpha^{FB}(\theta) - \int_{\underline{c}}^{c_0} H(c,\theta+\alpha^{FB}(\theta))dc \\ &= \gamma_0(\theta^*-\underline{\theta}) - \int_{\underline{c}}^{c_0} H(c,\theta^*)dc.\end{aligned}$$

Thus, $\tilde{\pi}(\theta,\overline{c},\overline{c})$ is a constant, independent of θ.

Therefore, $\pi(\hat{\theta},\theta)$ is a constant, for any report $\hat{\theta}$. As a result, IC_1 (7) is trivially satisfied. $\square$

6.2 Appendix B

Proof of Lemma A1: Fix the report $\hat{\theta}$. Recall the definition of $\delta(\hat{\theta})$ after equation (8). $\alpha(\hat{\theta},\theta) > 0$ if and only if $\theta < \delta(\hat{\theta})$; and $\alpha(\hat{\theta},\theta) = 0$ if and only if $\theta \geq \delta(\hat{\theta})$ (temporarily ignore the possibility that $\delta(\hat{\theta}) \notin [\underline{\theta},\overline{\theta}]$; see footnote 7). When $\theta < \delta(\hat{\theta})$, $\alpha(\hat{\theta},\theta) = \delta(\hat{\theta}) - \theta$, which is obviously a differentiable function of θ and $\frac{\partial\alpha(\hat{\theta},\theta)}{\partial\theta} = -1$. When $\theta > \delta(\hat{\theta})$, $\alpha(\hat{\theta},\theta) = 0$ so that it is also differentiable in θ and $\frac{\partial\alpha(\hat{\theta},\theta)}{\partial\theta} = 0$. As a result, $\alpha(\hat{\theta},\theta)$ is differentiable in θ except when $\theta = \delta(\hat{\theta})$. Notice that when $\delta(\hat{\theta}) > \overline{\theta}$, $\theta < \delta(\hat{\theta})$ for all $\theta \in [\underline{\theta},\overline{\theta}]$, so $\alpha(\hat{\theta},\theta)$ is differentiable in θ everywhere in $[\underline{\theta},\overline{\theta}]$. When $\delta(\hat{\theta})$ does not exist, $\alpha(\hat{\theta},\theta) = 0$, which is trivially differentiable in $[\underline{\theta},\overline{\theta}]$. Therefore, $\alpha(\hat{\theta},\cdot)$ is differentiable everywhere except possibly at one point. The continuity of $\alpha(\hat{\theta},\cdot)$ is obvious. $\square$

Proof of Lemma A2: If $\delta(\hat{\theta})$ does not exist, then $\alpha(\hat{\theta},\theta) = 0$ for all θ so that

$$\pi(\hat{\theta},\theta) = \tilde{\pi}(\hat{\theta},\overline{c},\overline{c}) + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta},c)H(c,\theta)dc,$$

which is differentiable in θ over $[\underline{\theta}, \overline{\theta}]$ by the Lebesgue's dominated convergence theorem. Moreover, its derivative is

$$\frac{\partial \pi(\hat{\theta}, \theta)}{\partial \theta} = \int_{\underline{c}}^{\overline{c}} p(\hat{\theta}, c) H_2(c, \theta) dc = \int_{\underline{c}}^{\overline{c}} p(\hat{\theta}, c) H_2(c, \theta + \alpha(\hat{\theta}, \theta)) dc,$$

which is obviously continuous in θ over $[\underline{\theta}, \overline{\theta}]$.

Now suppose $\delta(\hat{\theta})$ exists. We first show that $\pi(\hat{\theta}, \theta)$ is differentiable in $\theta \in [\underline{\theta}, \overline{\theta}]$. By Lemma A1, $\alpha(\hat{\theta}, \theta)$ is continuous and differentiable everywhere except at $\theta = \delta(\hat{\theta})$ (ignore the case in which $\delta(\hat{\theta}) > \overline{\theta}$, which is obviously true following the proof here). Therefore, for any $\hat{\theta}$, by the Lebesgue's dominated convergence theorem, $\pi(\hat{\theta}, \theta)$ is differentiable when $\theta \neq \delta(\hat{\theta})$. Now we only need to show that it is also differentiable at $\theta = \delta(\hat{\theta})$. Notice that when $\theta \geq \delta(\hat{\theta})$, $\alpha(\hat{\theta}, \theta) = 0$ so that $\pi(\hat{\theta}, \theta) = \tilde{\pi}(\hat{\theta}, \overline{c}, \overline{c}) + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta}, c) H(c, \theta) dc$. Therefore, by the dominated convergence theorem, the right derivative

$$\lim_{\theta \to \delta(\hat{\theta})^+} \frac{\pi(\hat{\theta}, \theta) - \pi(\hat{\theta}, \delta(\hat{\theta}))}{\theta - \delta(\hat{\theta})} = \lim_{\theta \to \delta(\hat{\theta})^+} \frac{\int_{\underline{c}}^{\overline{c}} p(\hat{\theta}, c)[H(c, \theta) - H(c, \delta(\hat{\theta}))] dc}{\theta - \delta(\hat{\theta})} = \int_{\underline{c}}^{\overline{c}} p(\hat{\theta}, c) H_2(c, \delta(\hat{\theta})) dc = \gamma_0.$$

When $\theta < \delta(\hat{\theta})$,

$$\begin{aligned} \pi(\hat{\theta}, \theta) &= -\gamma_0 \alpha(\hat{\theta}, \theta) + \tilde{\pi}(\hat{\theta}, \overline{c}, \overline{c}) + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta}, c) H(c, \theta + \alpha(\hat{\theta}, \theta)) dc \\ &= -\gamma_0 (\delta(\hat{\theta}) - \theta) + \tilde{\pi}(\hat{\theta}, \overline{c}, \overline{c}) + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta}, c) H(c, \delta(\hat{\theta})) dc. \end{aligned}$$

The left derivative

$$\lim_{\theta \to \delta(\hat{\theta})^-} \frac{\pi(\hat{\theta}, \theta) - \pi(\hat{\theta}, \delta(\hat{\theta}))}{\theta - \delta(\hat{\theta})} = \lim_{\theta \to \delta(\hat{\theta})^-} \frac{-\gamma_0 (\delta(\hat{\theta}) - \theta)}{\theta - \delta(\hat{\theta})} = \gamma_0.$$

Thus, $\pi(\hat{\theta}, \theta)$ is differentiable at $\theta = \delta(\hat{\theta})$.

Now we go on to show the other two properties. When $\theta < \delta(\hat{\theta})$, (16) holds with equality. Thus

$$\frac{\partial \alpha(\hat{\theta}, \theta)}{\partial \theta} (-\gamma_0 + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta}, c) H_2(c, \theta + \alpha(\hat{\theta}, \theta)) dc) = 0. \tag{23}$$

When $\theta > \delta(\hat{\theta})$, $\alpha(\hat{\theta}, \theta) = 0$ so that $\frac{\partial \alpha(\hat{\theta}, \theta)}{\partial \theta} = 0$. The above equality still holds.

Then, when $\theta \neq \delta(\hat{\theta})$,

$$\begin{aligned}\frac{\partial \pi(\hat{\theta},\theta)}{\partial \theta} &= -\gamma_0 \frac{\partial \alpha(\hat{\theta},\theta)}{\partial \theta} + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta},c) H_2(c,\theta+\alpha(\hat{\theta},\theta))(1+\frac{\partial \alpha(\hat{\theta},\theta)}{\partial \theta})dc \\ &= \int_{\underline{c}}^{\overline{c}} p(\hat{\theta},c) H_2(c,\theta+\alpha(\hat{\theta},\theta))dc.\end{aligned}$$

Since $\alpha(\hat{\theta},\theta)$ is continuous in θ, $\frac{\partial \pi(\hat{\theta},\theta)}{\partial \theta}$ is continuous when $\theta \neq \delta(\hat{\theta})$. Notice that $\lim_{\theta \to \delta(\hat{\theta})} \frac{\partial \pi(\hat{\theta},\theta)}{\partial \theta} = \gamma_0$, which is equal to the derivative of $\pi(\hat{\theta},\theta)$ at $\theta = \delta(\hat{\theta})$. Therefore, $\pi(\hat{\theta},\theta)$ is continuously differentiable in $[\underline{\theta},\overline{\theta}]$. $\square$

Proof of Lemma A3: Since Problem (O-R) does not involve any constraint linking different θ's, it can be solved in a pointwise manner. For any fixed θ, the problem is

$$\min_{\alpha \geq 0, p(\theta,c) \in [0,1]} \gamma_0 \alpha + \int_{\underline{c}}^{\overline{c}} p(\theta,c) \left[\tilde{J}(c,\theta,\theta+\alpha) - c_0\right] h(c,\theta+\alpha)dc$$

subject to

$$-\gamma_0 + \int_{\underline{c}}^{\overline{c}} p(\theta,c) H_2(c,\theta+\alpha)dc \leq 0, \text{ with equality if } \alpha > 0. \tag{24}$$

$$p(\theta,c) \text{ is decreasing in } c. \tag{25}$$

Call this Problem (O-R-E), which is equivalent to Problem (O-R). (We say that two optimization problems are *equivalent* if they have the same optimal solution(s).) There are two cases, depending on whether (24) is binding.

Case 1: (24) is binding at the optimum. In this case, the objective function can be simplified as follows:

$$\begin{aligned}&\gamma_0 \alpha + \int_{\underline{c}}^{\overline{c}} p(\theta,c) \left[\tilde{J}(c,\theta,\theta+\alpha) - c_0\right] h(c,\theta+\alpha)dc \\ = \quad &\gamma_0 \alpha + \int_{\underline{c}}^{\overline{c}} p(\theta,c)(c-c_0)h(c,\theta+\alpha)dc + \frac{1-G(\theta)}{g(\theta)} \underbrace{\int_{\underline{c}}^{\overline{c}} p(\theta,c) H_2(c,\theta+\alpha(\theta))dc}_{=\gamma_0} \\ = \quad &\gamma_0 \alpha + \int_{\underline{c}}^{\overline{c}} p(\theta,c)(c-c_0)h(c,\theta+\alpha)dc + \gamma_0 \frac{1-G(\theta)}{g(\theta)}.\end{aligned}$$

Thus, if (24) is binding at the optimum, Problem (O-R-E) is equivalent to Problem (O-R-E1):

$$\min_{\alpha \geq 0, p(\theta,c) \in [0,1]} \gamma_0 \alpha + \int_{\underline{c}}^{\overline{c}} p(\theta, c)(c - c_0) h(c, \theta + \alpha) dc$$

subject to

$$-\gamma_0 + \int_{\underline{c}}^{\overline{c}} p(\theta, c) H_2(c, \theta + \alpha) dc = 0. \tag{26}$$

$$p(\theta, c) \text{ is decreasing in } c. \tag{27}$$

Obviously, given Assumption 2, the efficient allocation

$$p^*(\theta, c) = \begin{cases} 1, \text{ if } c \leq c_0 \\ 0, \text{ if } c > c_0 \end{cases}$$

induces a positive effort, which is $\alpha^{FB}(\theta)$. In fact, the efficient allocation is actually optimal in Case 1.

To see this, suppose that (26) in Problem (O-R-E1) is dropped to form Problem (O-R-E1-R). The solution to Problem (O-R-E1-R) must be at least as good as that to Problem (O-R-E1). In Problem (O-R-E1-R), for any given $\alpha \geq 0$, the efficient allocation obviously minimizes the objective function. Therefore, under efficient allocation, the objective function of Problem (O-R-E1-R) becomes

$$\gamma_0 \alpha + \int_{\underline{c}}^{c_0} (c - c_0) h(c, \theta + \alpha) dc = \gamma_0 \alpha - \int_{\underline{c}}^{c_0} H(c, \theta + \alpha) dc.$$

The unique minimizer α^* of Problem (O-R-E1-R) satisfies

$$-\gamma_0 + \int_{\underline{c}}^{c_0} H_2(c, \theta + \alpha^*) dc = 0.$$

But when $p(\theta, c) = p^*(\theta, c)$, this is exactly constraint (26), which we dropped from Problem (O-R-E1) to form Problem (O-R-E1-R). Therefore, if (24) is binding at the optimum, the optimum has to be the efficient allocation.

Case 2: (24) is slack at the optimum. In this case, $\alpha = 0$, so Problem (O-R-E) is equivalent to

Problem (O-R-E2):[14]

$$\min_{p(\theta,c)\in[0,1]} \int_{\underline{c}}^{\overline{c}} p(\theta,c)(J(c,\theta)-c_0)h(c,\theta)dc$$

subject to

$$-\gamma_0 + \int_{\underline{c}}^{\overline{c}} p(\theta,c)H_2(c,\theta)dc < 0. \tag{28}$$

$$p(\theta,c) \text{ is decreasing in } c. \tag{29}$$

We show that Case 2 is impossible by showing that Problem (O-R-E2) does not have any solution. Suppose, to the contrary, that $\hat{p}(\theta,c)$ is a solution to Problem (O-R-E2). Let the set of allocations $p(\theta,c)$ that satisfy all of the constraints in Problem (O-R-E2) be S. For each $p(\theta,c)\in S$, denote $\tilde{c}(p(\theta,c)) = \sup\{c\in[\underline{c},\overline{c}] : p(\theta,c)=1\}$. Further define $\tilde{c}(\theta) = \sup\{\tilde{c}(p(\theta,c)) : p(\theta,c)\in S\}$. Then $\tilde{c}(\theta) < c_B(\theta)$. To see this, note that if $\tilde{c}(\theta)\geq c_B(\theta)$, then for any $\varepsilon>0$, there exists some $\tilde{p}(\theta,c)\in S$ such that

$$-\gamma_0 + \int_{\underline{c}}^{\overline{c}} \tilde{p}(\theta,c)H_2(c,\theta)dc > -\gamma_0 + \int_{\underline{c}}^{c_B(\theta)-\varepsilon} H_2(c,\theta)dc.$$

However, Assumption 2 implies that $-\gamma_0 + \int_{\underline{c}}^{c_B(\theta)-\varepsilon} H_2(c,\theta)dc > 0$ when ε is small enough. So there exists some $\check{p}(\theta,c)\in S$ such that

$$-\gamma_0 + \int_{\underline{c}}^{\overline{c}} \check{p}(\theta,c)H_2(c,\theta)dc > 0,$$

which contradicts (28).

Recall that we assume $\hat{p}(\theta,c)$ is a solution to Problem (O-R-E2). Therefore,

$$-\gamma_0 + \int_{\underline{c}}^{\overline{c}} \hat{p}(\theta,c)H_2(c,\theta)dc < 0.$$

Let $\hat{c}(\theta) = \sup\{c\in[\underline{c},\overline{c}] : \hat{p}(\theta,c)=1\}$. Then $\hat{c}(\theta)\leq\tilde{c}(\theta)<c_B(\theta)$. Consider the following allocation:

$$\hat{p}^*(\theta,c) = \begin{cases} \hat{p}(\theta,c), \text{ if } c\in[\underline{c},\hat{c}(\theta)), \\ 1, \text{ if } c\in[\hat{c}(\theta),\hat{c}(\theta)+\varepsilon], \\ \hat{p}(\theta,c), \text{ if } c\in(\hat{c}(\theta)+\varepsilon,\overline{c}]. \end{cases},$$

where $\varepsilon>0$ is small enough such that $\hat{c}(\theta)+\varepsilon<c_B(\theta)$ and $-\gamma_0+\int_{\underline{c}}^{\overline{c}}\hat{p}^*(\theta,c)H_2(c,\theta)dc<0$.

[14] Note that $\tilde{J}(c,\theta,\theta)=J(c,\theta)$.

It is easy to verify that $\hat{p}^*(\theta, c) \in S$. However, in Problem (O-R-E2), $\hat{p}^*(\theta, c)$ leads to a strictly lower objective function value than $\hat{p}(\theta, c)$ does, contradicting the optimality of $\hat{p}(\theta, c)$ in Problem (O-R-E2). To see this, notice that since $J(c, \theta) \leq c_0$ if and only if $c \leq c_B(\theta)$,

$$\int_{\underline{c}}^{\overline{c}} [\hat{p}^*(\theta, c) - \hat{p}(\theta, c)](J(c, \theta) - c_0)h(c, \theta)dc = \int_{\hat{c}(\theta)}^{\hat{c}(\theta)+\varepsilon} \underbrace{(1 - \hat{p}(\theta, c))}_{>0} \underbrace{(J(c, \theta) - c_0)}_{<0} h(c, \theta)dc < 0.$$

In sum, we have shown that at the optimum the moral hazard constraint (24) is binding, so the argument in Case 1 implies that the efficient allocation $p^*(\theta, c)$ is the (unique) solution to Problem (O-R-E). The corresponding investment level $\alpha^*(\theta)$ is obviously $\alpha^{FB}(\theta)$. $\square$

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